Spatio-temporal dynamics of substorms during intense geospace storms

J. Chen, A. S. Sharma, and X. Shao

Abstract: The nonlinear dynamical models of the coupled solar wind-magnetosphere system derived from observational data are used to yield efficient forecasts of the magnetospheric conditions. A correlated database of solar wind and magnetospheric time series data for the last solar cycle near its peak (year 2001) is compiled and used to model the magnetospheric dynamics under strong driving. The dynamical models of the magnetosphere during superstorms developed with this database are used to forecast the geospace storms of October-November 2003 and April 2002, and yields improved forecasts of the intense storms. A new technique which consider the contributions of the nearest neighbors weighted by factors inversely proportional to the distances in the reconstructed phase space yields better predictions, especially during the strongly driven periods. Also the time series data of the distributed observations are used to develop spatio-temporal dynamics of the magnetosphere using phase space reconstruction techniques. This nonlinear model is used to study the spatial structure of geomagnetic disturbances during intense geospace storms. The ground magnetometer data are from the two chains of stations: CANOPUS (13) and IMAGE (26). This new data set, with 1-minute resolution, is used to study the spatio-temporal structure, including the coupling between the high and mid-latitude regions. From the point of view of space weather the predictions of the spatial structure are crucial, as it is important to identify the regions of strong disturbances during intense geospace storms

Key words: Substroms,Nonlinear Dynamics ,Prediction, Space Weather.

1. Introduction

The solar wind-magnetosphere coupling is enhanced when the interplanetary magnetic field (IMF) turns southward, leading to geospace storms and substorms. The magnetosphere is a highly dynamic system under these conditions. The Earth's magnetosphere is a non-autonomous dynamical system, driven by the solar wind. Studies of the magnetospheric dynamics using models derived from the correlated database of the solar wind - magnetosphere system have enhanced our understanding of the complex behavior of the magnetosphere. The advantage of this approach is the ability to yield the dynamics, inherent in observational data, independent of modeling assumptions. There has been considerable progress in the modeling and forecasting of the solar wind-magnetosphere coupling as an input-output system by linear and nonlinear approaches.

The linear prediction filter technique was used to obtain the response time of the magnetosphere from the $AL - VB_s$ database [3][hereafter referred to as the BBMH dataset]. This database spans the period from November 1973 to December 1974 and has 2.5 min resolution. The response functions from this analysis have been used to interpret how the magnetospheric response to the solar wind driver with changes in the activity level, indicating nonlinearity. These response functions exhibited two time scales, corresponding to the directly driven and loading-unloading processes. The modeling of magnetospheric substorms as a low dimensional system using the time series data of the electrojet indices, *AL* or *AE*, to reconstruct its dynamics has shown its low dimensionality and the nonlinear nature of the magnetosphere [6] [7] [8] [17]. The reconstructed phase space show clear evidence that the dynamical system follows a pattern in the reconstructed phase space [9] [10]. This implies that the dynamics of the magnetosphere is predictable and this recognition has stimulated the study of forecasting substorms [18] and storms [14]. Vassiliadis [18] used the local-linear technique on the BBMH dataset, with the solar wind convective electric field VB_s as the input and the *AL* index as the output, and obtained good predictions. These predictions gave strong evidence that nonlinear models can be used to develop accurate and reliable forecasting tools for space weather. Recent studies using time series data have shown that the coherence on the global magnetospheric scale can be obtained by averaging over the dynamical scales. A model for the global features can be obtained by a mean field technique of averaging outputs corresponding to similar states of the system in the reconstructed phase space [12] [13]. With such a mean-field model, accurate iterative long-term predictions can be obtained, as the model parameters need not be changed during the prediction.

Recently, some dynamical models incorporating the spatial structure have been studied beyond the global indices. The successful standard nonlinear dynamic approach using the $AL - VB_s$ coupling has been generalized to consider the dynamical evolution of spatial structure of magnetic perturbation. Valdivia [15] [16] studied and modeled the evolution of the spatial structure of the middle and high latitude current structure by a set of mid- and high-latitude ground magnetometers distributed at different longitudes around the Earth, providing the representation of the effect of the currents at the ground. A 2D dynamical solar wind driven model for the evolution of the spatial structure of the mid-high latitude magnetic field perturbations was generated from IMAGE chain of magnetometers. The prediction model gives some new and interesting results.

During April 2002 and October-November 2003, nearly 2 years after the last solar maximum, three extremely big G5

Received 27 June 2006.

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geospace storms occurred, an extreme geomagnetic storm on the NOAA space weather scale that runs from G1 to G5. These three G5 extreme geomagnetic storms were driven by the solar wind with the southward IMF of -58.3 nT, -32.03 nT and -53.02 nT, measured by ACE, and these led to the *AL* index values of -2778 nT, -1851 nT and -2499 nT, respectively. These three intense geospace storms provide interesting opportunities for the study of nonlinear phase space reconstruction under extreme conditions. In order to model and predict such intense storms, a correlated database of the solar wind and magnetospheric variables of the year 2001, which is close to the peak period of 11-year solar cycle, was compiled [4].

To study the spatial structure as observed by the latitudinal chain of magnetometers, CANOPUS and IMAGE, the ground magnetometer measurements from 26 stations of IMAGE array and 13 stations of CANOPUS array for year 2002 with resolution of 1 minutes are compiled. The correlated solar wind input is VB_s, as in the earlier studies.

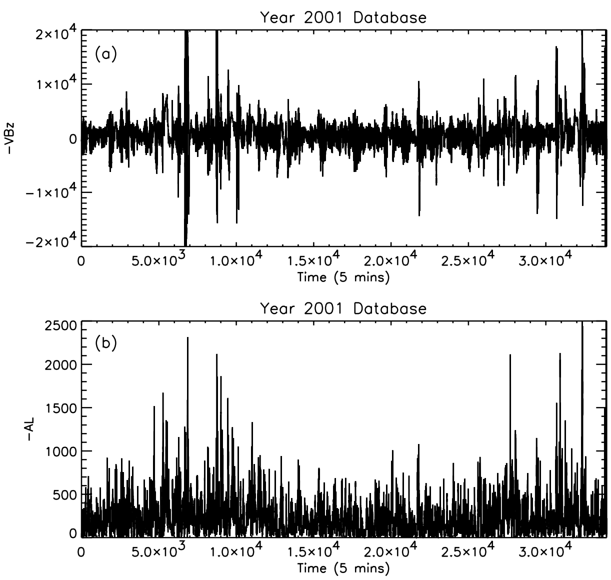

Fig. 1. The correlated solar wind induced electric field VB_z (panel a) and the auroral electrojet index AL (panel b) for 81 intense storm intervals during year 2001. The geomagnetic activity in these intervals during the peak of the last solar cycle is very high and correspond to strong driving by the solar wind.

2. Correlated Database of Solar Wind-Magnetosphere Coupling under Strong Driving

During the period of maximum solar activity, the magnetosphere is strongly driven and the year 2001 near the last solar maximum is chosen for compiling a database for such an epoch. This database contains solar wind flow speed *V*, the north-south component of the IMF B_z and the *AL* index for the 11 months of 2001 (January to November). The solar wind data for 2001 were compiled for a set of data intervals, each defined as any continuous data longer than 12 hours with no more than half-hour data gap. The dataset contains 81 intervals with periods 12 hours to 3 days long. During January-November 2001, there were 81 such data intervals containing 33931 data points at 5-min resolution, satisfying the above conditions. The correlated solar wind induced electric field VB_z and the auroral electrojet index *AL* for 81 intense storm intervals during year 2001 are shown on Figure 1. During this period of strong solar activity, intense substorms and storms were triggered with higher frequency. If we define a strong geomagnetic storm as having *Dst* less than -100 nT, we find that there are 12 such storms in 2001 compared with 4 such storms in 1995 and 1 in 1996. Thus the 2001 database is appropriate for studying the properties of geomagnetic activity during a solar maximum. The selected 81 events are separated into 3 activity levels by the average values of VB_s: medium ($\langle VB_s \rangle \leq$ 1500 nT km/s), high (1500 nT km/s $\leq \langle VB_s \rangle \leq$ 2500 nT km/s), and super ($\langle VB_s \rangle \geq$ 2500 nT km/s). To model a specific event, we choose the corresponding activity level to which it belongs and use it as a reference database.

During 2002-2003 there were three intense storms, occurring in April 2002, October 2003, and November 2003. The solar wind data from ACE through CDAWEB and the corresponding geomagnetic field index *AL* were compiled for these storms.

The magnetic perturbations from the 39 magnetometers of IMAGE and CANOPUS of year 2002 are used to visualize and predict the spatial evolution of the current systems. This database contain solar wind key parameters from ACE and magnetic perturbation from ground magnetometers with 1 minute resolution. We have both the magnetic perturbation H_x, geographic north, and H_y, geographic east, of the individual magnetometer. A valuebase, defined as the average value of the 15 quietest days in the whole year 2002, is subtracted from each component at each magnetometer.

We partition the dataset by mapping the magnetometer measurements in the universal time and the magnetic latitude to a 2D grid of magnetic local time and magnetic latitude λ [16]. Such mapping is possible because the perturbation is measured at the different location in the magnetosphere as the Earth rotates.

3. Nonlinear Dynamical Modeling Using Correlated Data

3.1. Input-Output Modeling of the Magnetosphere

The magnetosphere has been shown to exhibit the features of a nonlinear dynamical system, and its global features have been modeled by a few variables [2]. This remarkable property arises from the inherent property of phase space contraction in dissipative nonlinear systems. A dynamical input-output model can be constructed based on local-linear filters, which represent the relationship between the input $I(t)$ and the output $O(t)$ of the system.

The time delay embedding technique is an appropriate method for the reconstruction of the phase space and for obtaining its characteristic properties [5] [11]. In this technique, a m component phase vector X_i is constructed from this time series $x(t)$ as:

$$X_i = \{x_1(t_i), x_2(t_i), \cdots, x_m(t_i)\}, \quad (1)$$

where $x_k(t_i) = x(t_i - (k-1)T)$ and T is a time delay. If the embedding procedure is properly performed, the dynamical attractor underlying the observed time series will be completely unfolded, and the constructed states have one to one correspondence with the states in the original phase space. Appropriate values of the time delay T and the embedding dimension m can be obtained by using techniques such as the average mutual information and the correlation integral [1].

In an input-output model of the solar wind-magnetosphere system during substorms, the solar wind convective electric field VB_s is commonly used as the input and the geomagnetic activity index *AL* or *AE* as the output. Thus the input-output vector in the $2m$ dimensional embedding space can be constructed as

$$X_i = (I_1(t_i), \cdots, I_{M_I}(t_i), O_1(t_i), \cdots, O_{M_O}(t_i)), \tag{2}$$

where $M_I = M_O = m$. The 2m-dimensional state vector X_i at $t = t_1, t_2, \cdots t_N$, can now be used to construct a trajectory matrix for the dynamics of the system as:

$$X = \begin{bmatrix} I_1(t_1) & \cdots & I_m(t_1) & O_1(t_1) & \cdots & O_m(t_1) \\ x_1(t_2) & \cdots & I_m(t_2) & O_1(t_1) & \cdots & O_m(t_1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ I_1(t_N) & \cdots & I_m(t_N) & O_1(t_N) & \cdots & O_m(t_N) \end{bmatrix} \tag{3}$$

where N is the number of vectors. This $N \times 2m$ matrix contains all the dynamical features of the system contained in the data and yields its evolution in the reconstructed phase space.

3.2. Local-Linear and Weighted Mean Field Filters

The reconstructed phase space obtained from time series data has one-to-one correspondence with the states in the original phase space, thus making the prediction of the dynamical system possible. The main idea of this method is the use of the trajectories in the neighborhood of the state at time t to predict its location at the next time step. Knowing how the neighboring trajectories evolve, the location of the current state $x(t)$ at next time step $t + T$ can be predicted. The procedure is locally linear but is essentially nonlinear as the features of the neighboring trajectories are taken into account by considering a small neighborhood.

Given the current state, the states similar to it in the training set are selected as the first step. The similarity of the current state with any other state in the known data, which is referred to as the training set, is quantified by the Euclidean distance between them in the embedding space. The states within a specified distance of the current state are referred to as the nearest neighbors (NN). The prediction using the mean field approach have been used with the correlated BBMH database of solar wind and geomagnetic activity time series [12] [13].

$$O_{n+1} = \frac{1}{NN} \sum_{k=1}^{NN} X_k \tag{4}$$

In the mean field model, all the states in the specified neighborhood, the NN nearest neighbors, were used to obtain the center of mass by a simple averaging procedure. It is however the prediction can be improved if the states close to the current state contribute more than those farther away. Based on this recognition, a new filter based on the mean field filter is proposed to improve the accuracy and efficiency of predictions. This weighted filter takes into account the distance of the nearest neighbors. a set of weight factors **g** which depend inversely on the distances of each nearest neighbor from the mass center is introduced as

$$g_k = \frac{1}{d_k^2} / \sum_{i=1}^{NN} \frac{1}{d_i^2} \tag{5}$$

where d_i is the Euclidean distance of the *i*th nearest neighbor from the center of mass. The predicted output that includes this weighting of the neighbors is

$$O_{n+1} = \frac{1}{NN} \sum_{k=1}^{NN} X_k \bullet g_k, \tag{6}$$

The prediction accuracy is quantified by normalized mean square error (NMSE):

$$\eta = \frac{1}{\sigma_o} \sqrt{\frac{1}{N} \sum_{i=1}^{N} (O_i - O_i^*)^2}, \tag{7}$$

where O_i and O_i^* are the observed and predicted data, respectively, and σ_o is the standard deviation of O_i.

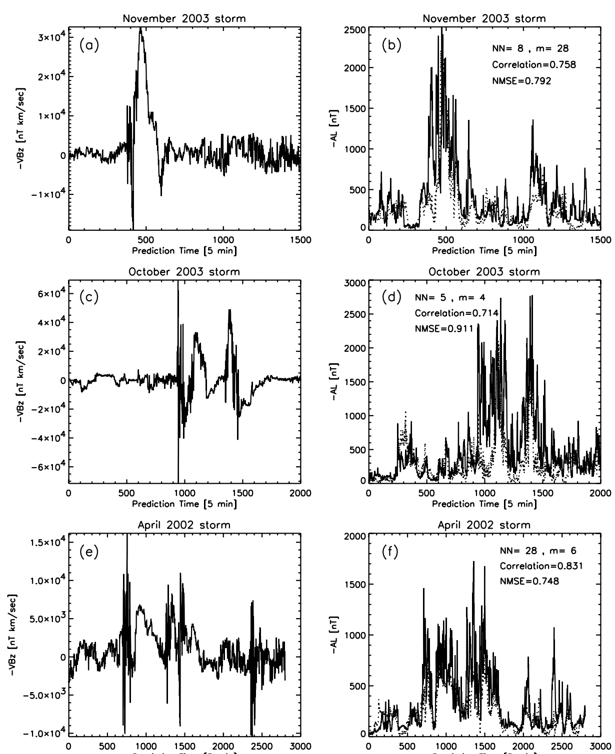

Fig. 2. The weighted mean-field predictions for storms: (a-b): November 19-26, 2003, (c-d): October 26-November 03, 2003, and (e-f): April 15-24, 2002. The left panel is VB_z, and the right panel is the real AL (solid line) and predicted AL (dotted line).

4. Modeling and Prediction during Superstorms

4.1. October-November 2003 and April 2002 Superstorms

The weighted mean field filter is used to model the solar wind-magnetosphere coupling during the superstorms of October-November 2003 and April 2002. In order to obtain the optimal nonlinear weighted mean field filter for superstorms, the following steps are adopted. First, the activity level of the solar wind driving is computed by averaging the southward component of VB_z. Then both the input (VB_z) and output (AL) of the time interval corresponding to the same activity level of the magnetospheric activity from the 2001 database are selected as the training set. For these three superstorms, the super level ($\langle VB_s \rangle$ $\geq$2500 nT km/s) of the 2001 database is selected. Second, using all of the selected data interval of input (VB_z) and its corresponding (AL) as a training set, the index AL is predicted for the superstorms using the weighted mean filter discussed above. The normalized mean square error (NMSE) is used to determine the optimal parameters for the prediction by comparing the predicted and actual AL. In this model, the time resolution (5 min) of the training set is chosen as the time delay T, and the other three free parameters are used to minimize the NMSE. The first two parameters are the embedding dimensions M_I and M_O, and as in the previous studies, we take $m = M_I = M_O$, which determines the vector length in the phase space to be $2m$. The third parameter is the number of nearest neighbors NN. A wide range of values of these parameters are used in the model to obtain the optimal predictions and these are shown in Figure 2. The solar wind convective electric filed ($-VB_z$) for these events are shown on Figure 2a, 2c and 2e. There is a sudden enhancement of the solar wind convective electric field in the early part of these events and this drives the geospace storms. The predicted and real AL are plotted in the panels (b), (d) and (f) of Figure 2. The solid lines represent the real AL and the dotted lines represent the predicted AL. Iterative predictions of the November 2003 storm were carried out for 7500 minutes (125 hours) with a minimum NMSE of 0.792 and the maximum correlation coefficient of 0.758. Also for the predictions of the October 2003 and April 2002 storms, yielded a minimum NMSE of 0.911 and 0.748, a maximum correlation coefficient of 0.714 and 0.831, respectively. In these figures the model output closely reproduces the large-scale variations of AL and captures some of the most abrupt changes. Also preceding the AL minima, there are sharp jumps, corresponding to the abrupt enhancements of the northward IMF. However, the southward IMF is the main driver of the geomagnetic storms, and it is not clear how well the model captures the effects of positive IMF enhancements.

In the earlier studies using the BBMH dataset [12] [13] [18], a major part of the dataset was used as the training set and the predictions were made for the remainder of the dataset. Consequently there were many similar states in the phase space. However for the two superstorms of 2003, it is hard to find so many similar big substorms in the available databases, such as that of year 2001. The nearest neighbor searches in these cases yields only a few states close to the superstorms. If we use a large number of nearest neighbors and a simple arithmetic averaging, the output of the model is smoothed over these and cannot capture the peak of the substorms. In such cases the weight factor g plays an important role and the averaging procedure yields improved predictions.

4.2. Comparison of Predictions using Bargatze [3] and Year 2001 databases

In order to compare the predictions using different databases as the training set, the storms of November 2003 are predicted using the BBMH database. To highlight the differences clearly, the periods of quiet and low activity before and after the main phase of the storms are neglected. The results of the storm of November 2003 are shown in Figure 3(a). It is clear that the peaks of AL cannot be predicted, mainly due to the absence of similar strong substorms in the BBMH database. The overall predictions have an NMSE of 0.847 and a correlation coefficient of 0.772. The predictions of for the same period using the year 2001 database and the combined database of year 2001 and BBMH are shown on Figure 3(b) and 3(c), respectively. A comparison of these predictions, Figure 3(a)-(c), shows the substantial improvement with the inclusion of the year 2001 database, either as the complete training set or as a part of a bigger training set. This is clearly due to the presence of many events in the year 2001 database similar to those in the November 2003 storm. In order to compare the predictability for different segments of the database, the November 2003 event was separated into smaller segments of 250 min or 50 data points each. The comparisons of the NMSE for the different segments are shown in Figure 3(d). It is clear that the NMSE for the data segments with large values of AL in the 2001 dataset are much smaller than those of the similar segments in the BBMH dataset.

The predictions and the NMSE for the storm of April 2002, a weaker storm compared to the November 2003 storm, are shown in Figure 4(a)-(c). The predictions are found to be almost the same when the three databases, viz. BBMH, year 2001, and the two combined, are used as the training sets. Also the NMSE values for 250 min intervals are shown in Figure 4 (d), and that NMSE have similar values in most of the segments.

In the case of the April 2002 storm, all the NMSE values obtained using different databases are similar, indicating that the BBMH and the 2001 databases yield similar predictions. However the 2001 database is a better choice for the October-November 2003 storms, as the comparisons in Figures 3 and 4 indicate. The remaining quieter periods of the October-November 2003 and the whole of April 2002 storms can be predicted very well using both the BBMH and Year 2001 databases as the training sets.

The analysis of the storms with different intensities and using different databases indicates that the geomagnetic response during the solar minimum and solar maximum periods have similar predictability. The 2001 and BBMH databases can thus be considered to complement each other. The combination of these two databases under different solar activities provides a comprehensive database for improved modeling and prediction of magnetospheric activity under a wide range of solar wind conditions.

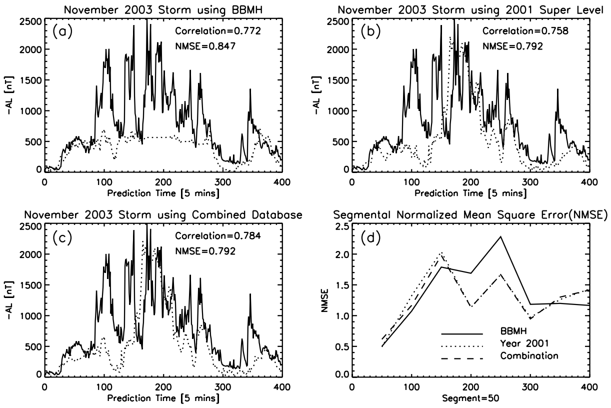

Fig. 3. The weighted mean field predictions on November 2003 storm using the BBMH, Year 2001 and combined databases. The solid line is real AL data, dotted line is predicted results. (a) BBMH database (b) Year 2001 database (c) Combined database (d) NMSE for 250-minute segments, the solid line represent BBMH database, dotted line represent Year 2001 database, dashed line represent combined database.

4.3. Spatial Structure of the High Latitude Magnetic Perturbations

The latitudinal chain of the magnetometers samples the spatial structure as the Earth rotates. So a full 2D dynamical model, driven by solar wind, of the spatial structure of the magnetic perturbations can be constructed. From such a 2D model, with a proper simultaneous solar wind selection, the localized solar wind-magnetic perturbation model can be established, and the prediction of locally region, instead of global indices, can be estimated.

The ground magnetic perturbations from 26 IMAGE and 13 CANOPUS are used to construct the 2D mapping during the April 2002 storm time. All of the station measurements are partitioned in a 2D grid that contains 24 hourly bins in magnetic local time and 26 or 13 bins, corresponding to the ground stations in IMAGE or CANOPUS array. Because of the simultaneous measurement of each magnetometer with same local time and different latitude, the high latitude magnetic perturbation can be seen on the average mapping both in magnetic latitude and local time as:

$$< H(\lambda, \xi) >= \frac{1}{N} \sum_{i=1}^{N} H(\lambda, t_i) \tag{8}$$

for H_x and H_y as shown at Figure 5 and Figure 6. Because the H_x and H_y are related to the east-west and south-north components of the current system. The Fig 5(a) and Fig 6(a) show a clear pattern of the westward and eastward currents during April 17-21, 2002, corresponding to the negative H_x in the midnight sectors and positive H_y in the noon sectors.

The basic structures of the high latitude magnetic perturbation are shown on these 2D averaged locally measurements. We are interested in the study this spatial dynamical system by considering the proper spatially dependent time delay between the onset solar wind and response of the magnetosphere response on different location.

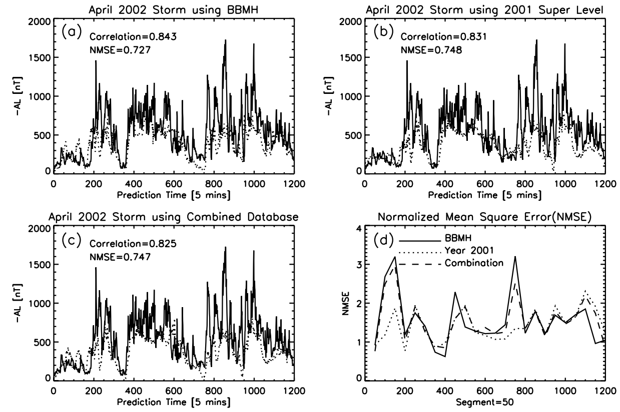

Fig. 4. The weighted mean field predictions on April 2002 storm using the BBMH, Year 2001 and Combined databases. The solid line is real AL data, dotted line is predicted results. (a) BBMH database (b) Year 2001 database (c) Combined database (d) NMSE for 250-minute segments, solid line represent BBMH database, dotted line represent Year 2001 database, dashed line represent combined database.

5. Conclusion

The modeling of magnetospheric response to strong driving by the solar wind is important not only for a better understanding of the solar wind - magnetosphere coupling and but also for developing our capability to forecast extreme conditions. During the last solar maximum there were many intense geospace storms and the existing models had limited success in forecasting these accurately. In order to develop better models and improve forecasting capability, a correlated database of the solar wind and the magnetospheric response is compiled for the year 2001 during the peak of the last solar cycle. In this database, the solar wind variable is the induced electric field and the magnetospheric response is the auroral electrojet index AL. This database is particularly well-suited for modeling using the phase space reconstruction techniques. The mean field approach to the modeling of the global magnetospheric dynamics [12] [13] is used to develop nonlinear dynamical models of the magnetospheric response from the year 2001 database. These predictions are then compared with the models based on the Bargatze [3] database, corresponding to a solar minimum period (1973 - 1974). The predictions for the big storms of October and November 2003 and April 2002 yields improved forecasts, especially for the intense storms.

The mean field approach has the advantage of yielding iterative predictions without having to fix model parameters, in particular the number of nearest neighbors NN and the dimension of the embedding space m [12] [13]. However during intense storms the number of similar events is usually small and this limits the ability to predict big events. In order to improve the predictability in such situations the mean field approach is modified by assigning weights to each of the nearest neighbors. These weights are inversely proportional to the square of the distance and leads to improvements in the predictions. The forecasting capability of the model is quantified in terms of a normalized mean square error (NMSE) computed from the predicted and actual AL values.

The two dimensional high latitude magnetic field perturb-

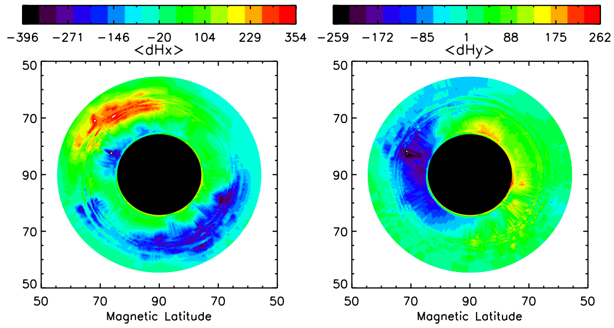

Fig. 5. The average value of H_x and H_y components measured by IMAGE in both magnetic latitude and local time over April 17-21, 2002.

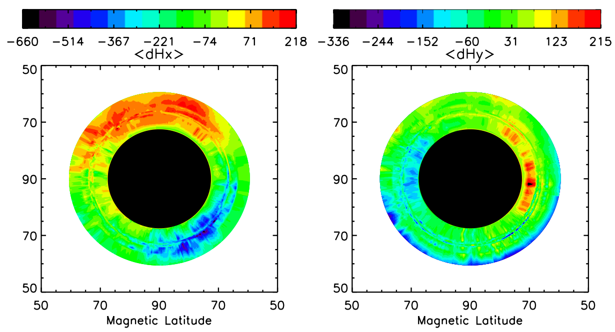

Fig. 6. The average value of H_x and H_y components measured by CANOPUS in both magnetic latitude and local time over April 17-21, 2002.

ations show the current structure of the magnetosphere. The solar wind driven model for these spatial variations can be derived from measurements of ground magnetometer chains after consider the proper time delay between the solar wind onset and proper magnetosphere response locations. With this model, we can study the spatial evolution of the current system as observed by multiple ground stations, and use it as a space weather forecasting tool.

Acknowledgments: We thank R. M. Skoug for the solar wind SWEPAM data and G. Lu for the AL data, and both for many fruitful discussions. We thank IMAGE and CANOPUS providing the data of ground magnetometers. The research is supported by NSF grants ATM-0318629 and DMS-0417800.

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