On the role of non-Maxwellian forms of distribution functions in the process of acceleration of auroral particles

N. O. Ermakova and E. E. Antonova

Abstract: Most theories of auroral particle acceleration are based on the suggestion of Maxwellian form of distribution function of accelerated particles. At the same time in most cases experimentally measured distribution functions are better described by kappa distribution. The formation of kappa distributions is connected with the action of relaxation processes in the turbulent magnetosphere in the conditions of the absence of collisions. Field-aligned acceleration of ionospheric ions leads to the appearance of particle population with plato-type distribution functions. The trapping of particles inside the region of acceleration also leads to the appearance of non-Maxwellian distributions. The model of auroral particle acceleration is developed taking into account the processes of modifications of distribution functions. The existence of conjugate regions of acceleration in the north and south hemispheres is suggested. It is shown that the kinetic treatment and the formation of non-Maxwellian distributions with reduced number of low energy particles gives the possibility to reanalyze the criteria of the formation of field-aligned jumps of electrostatic potential.

Key words: auroral acceleration, double layers, kappa distribution.

1. Introduction

Results of numerous observation at the auroral field lines demonstrate the existence of field-aligned potential drops (see the review [1]). However the processes of field-aligned potential drops formation are not clear till now. The most popular model is the model of the formation of double layers. Many theoretical studies of the processes of auroral particle acceleration were made under an assumption that the particle distribution functions have a Maxwellian form. Nevertheless, the collisionless character of magnetospheric plasma suggests the possibility of the existence of non-Maxwellian distribution functions.

Plasma sheet particle spectra can be approximated by a kappa distribution [2, 3]. The kappa distribution has the form

$$f(E) = \frac{n}{\pi^{3/2}} \frac{1}{\varepsilon_0^{3/2} k^{3/2}} \frac{\Gamma(k+1)}{\Gamma(k-1/2)} \cdot \left[1 + \frac{\varepsilon}{k\varepsilon_0}\right]^{-k-1}, \qquad (1)$$

where ε is the particle energy, ε_0 is the energy corresponding to the core thermal speed of the distribution, k is a parameter determining the high-energy power law index, n is the particle density, Γ is the gamma function. The kappa-function resembles a Maxwellian distribution at low energies, making a smooth transition into a power law tail at much higher energies. If the parameter $k \to \infty$ Maxwellian distribution is formed

$$f(E) = \frac{n}{\varepsilon_0^{3/2} \pi^{3/2}} \exp\left\{-\frac{\varepsilon}{\varepsilon_0}\right\}. \qquad (2)$$

Received 15 May 2006.

N. O. Ermakova. Skobeltsyn Institute of Nuclear Physics Moscow State University, Moscow, 119992, Russia
E. E. Antonova. Skobeltsyn Institute of Nuclear Physics Moscow State University, Moscow, 119992, Russia; Space Research Institute of RAS, Moscow, Russia

Hotter, more Maxwellian distributions may be identified as older in the sense of having undergone more velocity space diffusion.

The use of kappa distributions instead of Maxwellian distribution lead to the modifications of the theory of auroral particle acceleration. In this paper we consider the modifications of this theory by introducing the kappa distribution functions for population of hot magnetospheric electrons and ions.

2. Kappa distributions and classical double layer

The model of classical double layer considers the motion of cold current-carrying electrons and ions, together with warm electrons and ions, which are reflected by the double layer potential. The current-carrying electrons and ions are accelerated in opposite directions and gain energy from the double layer potential. The density of plasma in a double layer is much less, than outside of a double layer. Intensity of an electric field inside a layer is much larger then the intensity of a field in surrounding plasma. Occurrence of strong field-aligned fields in a double layer is caused by destruction of quasineutrality. The Poisson equation describes the distribution of potential Φ inside the double layer (Gaussian system of units is traditionally used)

$$\nabla^2\Phi = -4\pi e(n^i - n^e), \qquad (3)$$

where n^i, n^e are the ion and electron densities, e is the electron charge. In the one-dimensional double-layer all parameters depend only on one coordinate z along the magnetic field $\mathbf{B}$ and $d^2\Phi/dz^2 = 0.5(\Phi')^2/d\Phi$, $\Phi' = d\Phi/dz$. The equation (3) has the form

$$\frac{d}{d\Phi}(\Phi')^2 = -8\pi e\left[n^i(\Phi) - n^e(\Phi)\right] \qquad (4)$$

Necessary conditions for the formation of double layer with the potential drop Φ_k have the form

$$\begin{array}{lll} n^e > n^i & \text{if} & \Phi \to 0 \\ n^i > n^e & \text{if} & \Phi \to \Phi_k \end{array} \qquad (5)$$

The condition of quasineutrality outside the layer has the form

$$n^e(\Phi)_{\Phi=0} = n^i(\Phi)_{\Phi=0}, \quad n^e(\Phi)_{\Phi=\Phi_k} = n^i(\Phi)_{\Phi=\Phi_k}, \qquad (6)$$

and the condition of electric field vanishing outside the layer

$$\int_0^{\Phi_k} \left[n^e(\Phi) - n^i(\Phi)\right] d\Phi = 0 \qquad (7)$$

The distribution of charges and electric field inside the layer are determined by the analysis of particle motion inside the layer in the conditions of fixed distribution functions on the boundary of the layer. We shall consider laminar changes of potential (neglect the processes of distribution function relaxations inside the layer) and suggest the conservation of particle energy and magnetic moment.

Classical one-dimensional Langmuir double-layer suggests the existence of two cold and two hot populations of ions and electrons. If j_i is a beam of cold ions and j_e is a beam of cold electrons electron and ion concentrations are

$$\begin{array}{l} n^{ec} = j_e / \left[V_{0e}^2 + 2e\Phi/m_e\right]^{1/2}, \\ n^{ic} = j_i / \left[V_{0i}^2 + 2e(\Phi_k - \Phi)/m_i\right]^{1/2} \end{array} \qquad (8)$$

where V_{0e} and V_{0i} are electron and ion velocities before the acceleration, m_e and m_i are the electron and ion masses. It is considered that $\Phi = 0$ on the cathode part of the layer, $\Phi = \Phi_k$ on the anode part of the layer. If hot ions have Maxwell distribution function with temperature T_i and hot electrons with temperature T_e

$$\begin{array}{l} n^{ih} = n_0^{ih} \exp\left[-e\Phi/T_i\right], \\ n^{eh} = n_0^{eh} \exp\left[-e(\Phi_k - \Phi)/T_e\right]. \end{array} \qquad (9)$$

The condition (5) then leads to the Bohm-Block criterion for cold beams (see [4])

$$\begin{array}{l} m_e V_{0e}^2 > T_i, \\ m_i V_{0i}^2 > T_e. \end{array} \qquad (10)$$

Using of kappa distribution function leads to the modification of the criterion (10). For kappa distribution of hot ions with parameters ε_0^{ih}, k^{ih} and hot electrons with parameters ε_0^{eh}, k^{eh}

$$\begin{array}{l} n^{ih} = n_0^{ih} \left(1 + \dfrac{e\Phi}{k^{ih}\varepsilon_0^{ih}}\right)^{-k^{ih}+1/2}, \\ n^{eh} = n_0^{eh} \left(1 + \dfrac{e(\Phi_k - \Phi)}{k^{eh}\varepsilon_0^{eh}}\right)^{-k^{ih}+1/2}, \end{array} \qquad (11)$$

and relations (10) are changed to

$$\begin{array}{l} m_e V_{0e}^2 > \varepsilon_0^{ih} \left(1 - \dfrac{1}{k^{ih}}\right), \\ m_i V_{0i}^2 > \varepsilon_0^{eh} \left(1 - \dfrac{1}{k^{eh}}\right). \end{array} \qquad (12)$$

The comparison of relations (11) and (12) shows that the existence of non-Maxwellian tails of distribution functions leads to increase of ion and electron beam energies necessary for the formation of classical double layer. This limitation is stronger for more young distribution functions with smaller k.

The condition of the existence of stationary strong double layer has the form (Langmuir criterion):

$$j_e/j_i = \sqrt{m_i/m_e} \qquad (13)$$

where j_e, j_i are the electron and ion particle fluxes, m_e, m_i are the electron and ion masses. This condition is strongly modified if we take into account the existence of trapped populations of particles and kappa-form of distribution functions.

3. Kappa distribution and kinetic double layer

The collisionless character of magnetospheric particle motion leads to the development of kinetic theory of double layer formation (see [5]). The existence of double layers in the conjugate hemispheres and the possibility of the existence of non-Maxwellian population of hot ions produced by depredated ion beams accelerated by field-aligned potential drops from both hemispheres are taken into account in [6]. A water bag distribution function have been selected for this ion population, trapped in the equatorial plane. The contribution of the population of secondary electrons trapped between field-aligned potential drop and the ionosphere was analyzed in [7]. It was shown that the existence of trapped populations of electrons and ions gives the possibility to overcome the limitations given by Bohm-Block criterion.

We consider the model of electron and ion acceleration for the case of the existence of 3 ion populations and 3 electron populations: cold ionospheric electrons, hot magnetospheric ions described by kappa distribution, trapped near the equatorial plane ions of ionospheric origin accelerated in the field-aligned potential drop with the value Φ_k, cold electrons of ionospheric origin, hot electrons of magnetospheric origin described by kappa distribution, warm secondary electrons of magnetospheric origin trapped between the magnetic and electrostatic mirrors. The acceleration takes place in the region where $B = B_c$. All particles which can move up to ionospheric altitudes where $B = B_k$ are absorbed. However the contribution of particles lost in the ionosphere is comparatively small if the acceleration takes place at great geocentric distances (as it is ordinarily observed). Therefore we shall neglect such particles in the first approximation and consider the potential jump leading to great changes of particle concentrations. We shall also consider that the criterion (5) is satisfied due to the existence of trapped populations of ions and electrons.

The condition (6) leads to the expression

$$\begin{array}{l} n_0^{em} + n^{ei}\big|_{\Phi=0} = n_0^{im} + n_t^{ii} + n^{ii}\big|_{\Phi=0} \\ n_0^{ei} + n^{em}\big|_{\Phi=\Phi_k} + n_t^{em} = n_0^{ii} + n^{im}\big|_{\Phi=\Phi_k} \end{array} \qquad (14)$$

where the first upper indexes correspond to type of particle (e is for electrons, i is for ions), the second upper index corresponds to the type of population (i is for the ionospheric population,

m is for the magnetospheric population), symbol t shows the trapped particle population.

Taking into account the conservation of energy ε and magnetic moment μ in the process of acceleration, we shall use the electron and ion distribution functions in the form (1). Then

$$f d\mathbf{v} = \frac{n_0^m}{2} \frac{\Gamma(k^m+1)}{\Gamma(k^m-1/2)} \frac{B}{(\varepsilon_0^m)^{3/2}\pi^{1/2}(k^m)^{3/2}} \times \times \left[1+\frac{\varepsilon}{k^m\varepsilon_0^m}\right]^{-k^m-1} \frac{d\varepsilon\, d\mu}{(\varepsilon - e\Phi - B\mu)^{1/2}} \quad (15)$$

where $k^m = k^{im}$ and $\varepsilon_0^m = \varepsilon_0^{im}$ for hot magnetospheric ion population, $k^m = k^{em}$ and $\varepsilon_0^m = \varepsilon_0^{em}$ for hot magnetospheric electron population, e is the charge of particle.

The calculation of ion density inside the acceleration region leads to the expression

$$n^{im} = n_0^{im}\left(1+\frac{e\Phi}{k^{im}\varepsilon_0^{im}}\right)^{-k^{im}+1/2} \quad (16)$$

Ion distribution is isotropic. Expression (16) gives $n^{im}\big|_{\Phi=\Phi_k}$ if $\Phi = \Phi_k$.

The calculation of the density of hot magnetospheric electrons inside the double-layer leads to the expression

$$n^{em} = = n_0^{em}\left\{\left(1-\frac{e\Phi}{k^{em}\varepsilon_0^{em}}\right)^{-k^{em}+1/2} - \frac{4}{3\pi^{1/2}}\frac{\Gamma(k^{em}-1/2)}{\Gamma(k^{em}-1)} \times \times \left(1-\frac{e\Phi}{k^{em}\varepsilon_0^{em}}\right)^{-k^{em}-1}\left(\frac{e\Phi}{k^{em}\varepsilon_0^{em}}\right)^{3/2} \times \times {}_2F_1\left(k^{em}+1, \frac{3}{2}, \frac{5}{2}, \frac{e\Phi}{k^{em}\varepsilon_0^{em}-e\Phi}\right) - \frac{1}{k^{em}}\sqrt{\frac{e\Phi}{k^{em}\varepsilon_0^{em}}}\right\} \quad (17)$$

where we use the expression

$$\int_0^u \frac{x^{\mu-1}dx}{(1+\beta x)^\nu} = \frac{u^\mu}{\mu}{}_2F_1(\nu,\mu,1+\mu,-\beta u).$$

Expression (17) gives $n^{em}\big|_{\Phi=\Phi_k}$ if $\Phi=\Phi_k$.

Temperature of ionospheric ions is an order of magnitude lower than the multiplied on the electron charge field aligned potential drop. Therefore it is possible to neglect the term in the expression (14). We have for cold beam of ionospheric ions

$$n^{ii}\big|_{\Phi=0} = n^{ii}(\Phi_k, B_c)\left(\frac{m_i(V_0^{ii})^2}{2e\Phi_k}\right)^{1/2}, \quad (18)$$

where V_0^{ii} is the beam velocity.

Condition (7) gives

$$n_0^{em}k^{em}\varepsilon_0^{em}\left\{\left(\frac{1}{k^{em}-3/2}\left(1-\frac{e\Phi_k}{k^{em}\varepsilon_0^{em}}\right)^{-k^{em}+3/2} - 1\right) - -\left(\frac{e\Phi_k}{k^{em}\varepsilon_0^{em}}\right)^{5/2} \times \frac{8}{15\pi^{1/2}}\left(1-\frac{e\Phi_k}{k^{em}\varepsilon_0^{em}}\right)^{-k^{em}-1} \times \times {}_2F_1\left(k^{em}+1, \frac{5}{2}, \frac{7}{2}, \frac{e\Phi_k}{e\Phi_k - k^{em}\varepsilon_0^{em}}\right) - -\frac{4}{3\pi^{1/2}}\frac{\Gamma(k^{em}-1)}{\Gamma(k^{em}-1/2)}\frac{1}{k^{em}}\left(\frac{e\Phi_k}{k^{em}\varepsilon_0^{em}}\right)^{5/2}\right\} + + n_0^{ei}\frac{k^{ei}\varepsilon_0^{ei}}{3/2-k^{ei}}\left\{\left(1+\frac{e\Phi_k}{k^{ei}\varepsilon_0^{ei}}\right)^{-k^{ei}+3/2} - 1\right\} + + \int_0^{e\Phi_k} n_t^{em}(\Phi, B_c)e\, d\Phi = = n_0^{ii}(2e\Phi_k m_i)^{1/2}V_0^{ii} + \frac{n_0^{im}k^{im}\varepsilon_0^{im}}{k^{im}-3/2} \times \times\left\{\left(1+\frac{e\Phi_k}{k^{im}\varepsilon_0^{im}}\right)^{-k^{im}+3/2} - 1\right\} + \int_0^{e\Phi_k} n_t^{ii}(\Phi, B_c)e d\Phi \quad (19)$$

Analyzing (19) and comparing it with (13) it is possible to see that the condition of the existence of stationary strong double layer is greatly changed in the case of the double layer in the magnetic trap. It is interesting to mention that the appearance of such kind of structures leads to great jumps of plasma density.

4. Conclusions and discussion

The problem of the acceleration of auroral electrons by field-aligned potential drops is considered as a part of the problem of auroral particle acceleration and therefore as a part of the problem of magnetospheric substorm. The most popular model of such acceleration is the model of the formation of double layers [4–25]. But this problem in the majority of cases is analyzed in the suggestion of Maxwellian form of distribution functions. At the same time kappa distributions are formed in the collisionless magnetospheric plasma. Taktakishvili *et al.* [26] have shown that the results of INTERBALL/Tail probe observations of kappa distribution functions in the geomagnetic tail are well described by the process of acceleration by the inductive electric fields. This finding is supported by the results of [27]. Milovanov and Zelenyi [28] associate the origin of kappa distributions with the macroscopic ordering of the system. They prove that the canonical distribution corresponding to the Tsallis definition of entropy coincides with the kappa distribution. In accordance with [29] hotter, more Maxwellian distributions may be identified as older in the sense of having undergone more velocity space diffusion. Therefore it is quite interesting to reanalyze the conclusions of double-layer theories in the case of kappa distribution. We show that the real modifications appear as in the case of classical double layers as in the case of double layer in the magnetic trap. The later mainly leads to drops of particle densities. The condition of the existence of stationary double layer is obtained in both cases. The obtained relations can be used in the analysis of the results of auroral particle observations during substorms.

Acknowledgments

The research was supported by RFBR grant 05-05-64394-a and INTAS grant 03-51-3738.

References

1. Paschmann, G., Haaland, S., and Treumann, R., Auroral plasma physics, *Space Sci. Rev.*, *103*(1–4), 1–485, 2002.
2. Sarris, E. T., Krimigis, S. M., Lui, A. T. Y., Ackerson, K. L. Frank, L. A., and Williams, D. J., Relationship between energetic particles and plasmas in the distant plasma sheet, *Geophys. Res. Lett.*, *8*(3), 349–352, 1981.
3. Christon, S. P., Williams, D. J., Mitchell, D. G. *et al.*, Spectral characteristics of plasma sheet ion and electron population during undisturbed geomagnetic conditions, *J. Geophys. Res.*, *94*(10), 13,409–13,424, 1989.
4. Kan, J. R., Towards a unified theory of discrete auroras, *Space Sci. Rev.*, *31*(1), 71–117, 1982.
5. Antonova, E. E., and Tverskoy, B. A., Nature of electron precipitation band of the inverted V type and of Harang discontinuity in the evening sector of the auroral ionosphere, *Geomagnetism and Aeronomy (English translation)*, *15*(1), 85–90, 1975.
6. Antonova, E. E., Large-scale double layers, *Geomagnetism and Aeronomy (English translation)*, *19*(5), 586–589, 1979.
7. Kan, J. R., and Lee, L. C., On the auroral double layer criterion, *J. Geophys. Res.*, *85*(A2), 788–790, 1980.
8. Alfvén, H., and Carlqvist, P., Currents in the solar atmosphere and a theory of solar flares, *Solar Physics*, *1*(2), 200–228, 1967.
9. Block, L. P., Potential double layers in the ionosphere, *Cosmic Electrodynamics*, *3*(3), 352–376, 1972.
10. Carlqvist, P., On the formation of double layers in plasma, *Cosmic Electrodynamics*, *3*(3), 377–388, 1972.
11. Carlqvist, P., On the physics of relativistic double layers, *Astrophysics and Space Science*, *87*, 21–39, 1982.
12. Kan, J. R., Energization of auroral electrons by electrostatic shock waves, *J. Geophys. Res.*, *80*(16), 2089–2095, 1975.
13. Kan, J. R., Lee, L. C., and Akasofu, S. I., Two-dimentional potential double-layers and discrete aurora, *J. Geophys. Res.*, *84*(A8), 4305–4315, 1979.
14. Swift, D. W., A mechanism for energizing electrons in the magnetosphere, *J. Geophys. Res.*, *70*(13), 3061–3073, 1965.
15. Swift, D. W., On the formation of auroral arcs and acceleration of auroral electrons, *J. Geophys. Res.*, *80*(16), 2096–2108, 1975.
16. Swift, D. W., Steinback-Nielsen, H. C., and Hallinan, T. Y., An equipotential model for auroral arcs 1, *J. Geophys. Res.*, *81*(22), 3931–3934, 1976.
17. Swift, D. W., An equipotential model for auroral arcs 2. Numerical solutions, *J. Geophys. Res.*, *81*(22), 3955–3943, 1976.
18. Goertz, C. K., and Joyce, G., Numerical simulation of the plasma double layers, *Astrophys. Space Sci.*, *32*(1), 165–173, 1975.
19. Hubbard, R. F., and Joyce, G., Simulation of auroral double layers, *J. Geophys. Res.*, *84*(A8), 4297–4304, 1979.
20. Singh, N., and Thiemann, H., Some features of inverted V events as seen from simulated double layers, *Geophys. Res. Lett.*, *75*(10), 737–740, 1980.
21. Sato, T., and Okuda, H., Ion-acoustic double layers, *Phys. Rev. Lett.*, *44*(11), 740–743, 1980.
22. Sato, T., and Okuda, H., Numerical simulations on ion acoustic double layers, *J. Geophys. Res.*, *86*(A5), 3357–3368, 1981.
23. Singh, N., and Schunk, R. W., Dynamical features of moving double layers, *J. Geophys. Res.*, *87*(A5), 3561–3586, 1982.
24. Borovsky, J. E., Joyce, G., Numerically simulated two-dimensional auroral double layers, *J. Geophys. Res.*, *88*(A4), 3116–3126, 1983.
25. Bespalov, P. A., and Mizonova, V. G., Charged particle acceleration by electrostatic turbulent structures in regions with field-aligned electric current in the auroral magnetosphere, *Phys. Chem. Earth*, *26*(1–3), 113–119, 2001.
26. Taktakishvili, A. L., Zelenyi, L. M., Lutsenko, V. N., Kudela, K., On the spectra of energetic particles in the tail of the magnetosphere of the Earth, *Kosm. Issl.*, *36*(2), 282–291, 1998.
27. Pisarenko, N. F., Budnik, E. Yu., Ermolaev, Yu. I., Kirpichev, I. P., Lutsenko, V. N., Morozova, E. I., Antonova, E. E., The ion differential spectra in outer boundary of the ring current: November 17, 1995 case study, *J. Atmosp. and Solar-terrestrial Phys.*, *64*(5–6), 573–583, 2002.
28. Milovanov, A. V., and Zelenyi, L. M., Functional background of the Tsallis entropy: coarse-grained systems and kappa distribution functions, *Nonlinear Processes in Geophysics*, *7*(2), 211–221, 2000.
29. Collier, M. R., Evolution of kappa distributions under velocity space diffusion: A model for the observed relationship between their spectral parameters, *J. Geophys. Res.*, *104*(A12), 28,559-28,564, 1999.