Nonlinear stability of the near-earth plasma sheet during substorms

P. Dobias, J. A. Wanliss, and J. C. Samson

Abstract: We analyze a nonlinear stability of the near-Earth plasma sheet via a Grad-Shafranov equilibrium constrained by CANOPUS data. Using a stability analysis based on comparison of various orders in a Taylor expansion of the potential energy density, we demonstrate that an occurrence of field line resonances followed by a development of a Kelvin-Helmholtz instability at about 10 Re causes the near-Earth plasma sheet to become unstable minutes before the onset.

Key words: Substorms, Kelvin-Helmholtz, Ballooning.

1. Introduction

In the present work we address a nonlinear stability of the near-Earth plasma sheet during the substorm onset assuming a presence of the Kelvin-Helmholz (KH) instability. [12] outline a possible sequence of events that take place during the substorm expansion phase. This sequence assumes the initiation of the expansion phase near Earth. While there are other opinions we believe that the near-Earth initiation provides the simplest explanation for the observed sequence of events [8].

We use a stability analysis approach suggested by [9], improved further by [4]. This stability method can be summarized as follows. A plasma equilibrium is modeled using the Grad-Shafranov equation constrained by CANOPUS observations [4] to ensure that our tested configurations are relevant for the substorm event being analyzed. We are not modeling a transition between equilibrium states, rather, we calculate each configuration as a separate equilibrium based on observations. In the second step we define plasma plasma perturbation in the form of a displacement connecting Lagrangian and Eulerian description $\hat{\boldsymbol{x}}(\boldsymbol{x},t) = \boldsymbol{x} + \boldsymbol{\xi}(\boldsymbol{x},t)$. All other perturbed quantities are expressed in terms of the displacement. Then this plasma displacement is used to calculate expansion terms in the potential energy density. Comparison of the terms yields the stability properties of the system [9]. If the second order term is dominant, the system is well described by linear approximation. If the third order is dominant, the system is explosively unstable [9, 7]. The dominant fourth order term means that the system is nonlinearly stable [7]. This method allows us to estimate a possible maximum growth of the instability before it is saturated by nonlinear effects [5].

The above method is then used to analyze the nonlinear stability of the near-Earth plasma sheet during the February 9, 1995 substorm. We extend previous work of [6] by considering a possible development of Kelvin-Helmholtz instabilities due to a strong velocity shear caused by field line resonances [14]. We analyze changes of the stability properties of the plasma sheet due to the presence of a vortex, and also discuss a possible influence of the stability on the further development of this vortex. Our stability analysis is based on the ideal MHD, and it does not include influences of various other factors such as diffusion, Larmor radius effects, or azimuthal pressure gradient.

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2. Equilibrium Magnetospheric Model

For the stability analysis we use a Grad-Shafranov equilibrium in the form [12]

$$\psi(r,\theta) = 2\pi \frac{M \sin^2\theta}{r}\Big[1 + \frac{1-\alpha}{2}\Big(\frac{r}{R_X}\Big)^3 + \frac{\alpha}{4}\Big(\frac{r}{R_X}\Big)^5\Big]. \quad (1)$$

The radius r, polar angle θ, and azimuthal angle ϕ are spherical coordinates. M is the dipolar moment, and α and R_X are parameters characterizing pressure gradient and position of the X line. The solution (1) is a reasonable approximation as far as the position of the x-line, but breaks down at large distances. Its advantage is that it allows a relatively simple correlation with experimental observations via adjustment of the two parameters α and R_X using a position of the proton isotropy boundary and the position of the red emissions obtained from CANOPUS ground based observations [16, 17].

We use the distribution of auroral luminosity, from meridian scanning photometer data, to gain information about the nature and location of various plasma boundaries in the magnetotail. As shown previously by [16] and [17] meridian scanning photometers are excellent tools for investigating precipitation of charged particles in the auroral ionosphere and can quite easily be used to constrain magnetospheric magnetic field models.

The idea that we exploit in this paper relies on nonconservation of the first adiabatic invariant when magnetic field variations occur on the scale of a particle gyroradius. A measure of nonconservation is determined by the square root of the ratio of the magnetic field line radius of curvature to the particle gyroradius [2]. Theoretical effort by [18] suggests that the transition between the taillike and dipolelike field configurations occurs where the above ratio equals 3. In addition to the need for a magnetic field model, one can only find these locations if the energy of the precipitating particles is known. In the present work, the energies of the precipitating particles are determined directly from the equilibrium magnetotail model and are not a free parameter.

A new constraint is obtained from the model in that one of the model parameters is directly related to the location of the last closed magnetic field line. This boundary can also be obtained from ground-based photometer data [1]. They demonstrated that the poleward border of 630.0 nm optical emissions is very close to the transition between open and closed field lines in the dusk-midnight sector. In this present paper we follow the same methodology to determine the ionospheric location of the last closed magnetic field line.

Once an event has been selected for analysis, we calculate the equatorward edge of the 486.1 nm proton aurora, and the poleward edge of the 630.0 nm electron aurora as described above. The two free parameters in the magnetic field model are varied via the Levenburg-Marquardt nonlinear optimization method to minimize the sum of the square of the error between the model boundaries and the boundaries obtained from the photometer data.

We analyze the stability of the near-Earth plasma sheet during the February 9, 1995 event. The substorm onset occurred approximately at 4:37 UT, with the first weak disturbances appearing between 4:30 and 4:35 UT [6]. We performed stability tests for three times around the onset. The first stability test was performed at 4:30 UT, just before any significant disturbances started occurring. The second test was performed for the 4:35 UT configuration, at the time just prior to onset. The last test we have performed corresponded to the beginning of the recovery phase at 4:40 UT. The last test was performed to investigate changes of energy balance that the onset might have caused. Plasma pressure and magnetic field calculated in the equatorial plane are shown in Figures 1 and 2. The pressure gradient increases and the region of maximum pressure is moving earthward. The value of plasma β at 10 Re is 14 for the 4:30UT, later it increases to 40 for 4:35 UT, and then drops to 10 for 4:40 UT configuration. The magnetic field lines are being stretched, this is marked by a drop of magnetic field in the near-Earth region.

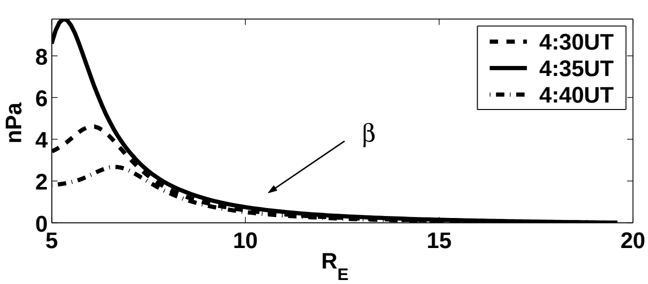

Fig. 1. Plasma pressure in the equatorial plane for 4:30, 4:35 and 4:40 UT configurations.

3. Plasma Displacement for K-H Instability

To perform the stability analysis we needed to choose is the specific form of plasma displacement. Here we extend the work of [6] where the authors assumed the plasma displacement corresponding to field line resonances,and have conclusively shown that approximately 2 minutes prior to onset the plasma sheet becomes nonlinearly unstable. [14] showed that a presence of the 180°-phase shift in the velocity will lead to

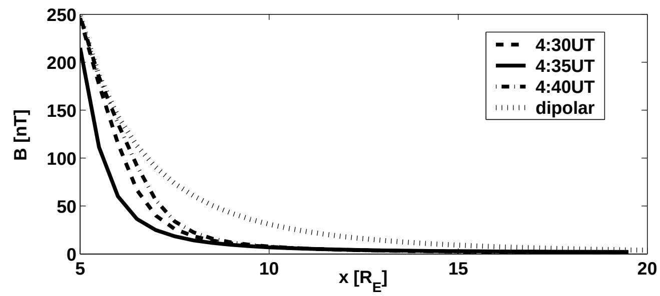

Fig. 2. Magnetic field in the equatorial plane for 4:30, 4:35 and 4:40 UT configurations. Dotted line shows dipolar field for the reference.

a development of the Kelvin-Helmholtz instability, that can be further coupled to a ballooning modes. Therefore in this paper we study the influence of the development of K-H instability on the overall stability of the near-earth plasma sheet.

We assume azimuthal symmetry for our analysis, and performed the stability analysis around 10 $\mathrm{R_E}$, in the region where FLRs are likely to occur [12]. This location further corresponds to a possible location of the processes responsible for forming the breakup arc [13]. The development of the K-H instability along the azimuthal position of the resonance will have a two fold effect on the dynamics of the displacement. First, it will increase the gradient of the plasma displacement due to development of a vortex structure. The second effect of the wrapping is the limitation of the growth of the magnitude of the displacement. If the K-H instability is coupled to a ballooning type instability the growth can continue as the shear flow-ballooning instability [14].

We use a fluid approximation to model development of the K-H instability. This is justified by the fact that our analysis is restricted to the equatorial plane where the magnetic field is perpendicular to the velocity field and thus there is no wrapping of magnetic field lines present. To model the K-H instability into its nonlinear stage we use a semi-analytical approach first presented by [10]. It is based on approximating the surface of discontinuity by a series of elementary vortices. Figure 3 shows development of the surface of discontinuity including the direction of velocity vectors. The distance in x and y directions is normalized in terms of wavelength λ of the instability. The wrapping of the surface of discontinuity is clearly visible at the later stages of the development. The magnitude of the displacement is approximately 0.2λ. The ambient velocity is scaled as ± 1.

Once we calculate temporal development of the K-H vortex, we are ready to model coupling between FLR's and the K-H instability. For simplicity, we can assume that the resonance introduces the velocity shear $\pm|u_{\phi FLR}|$ along the azimuthal direction. Since we are dealing with the dynamics close to the midnight plane, we can approximate the plasma sheet by a box model, with positive x in the tailward direction and y in the azimuthal direction. Then the surface of discontinuity is in the y-direction at the position of resonance, x_r. The necessary radial perturbation is introduced by the resonance itself.

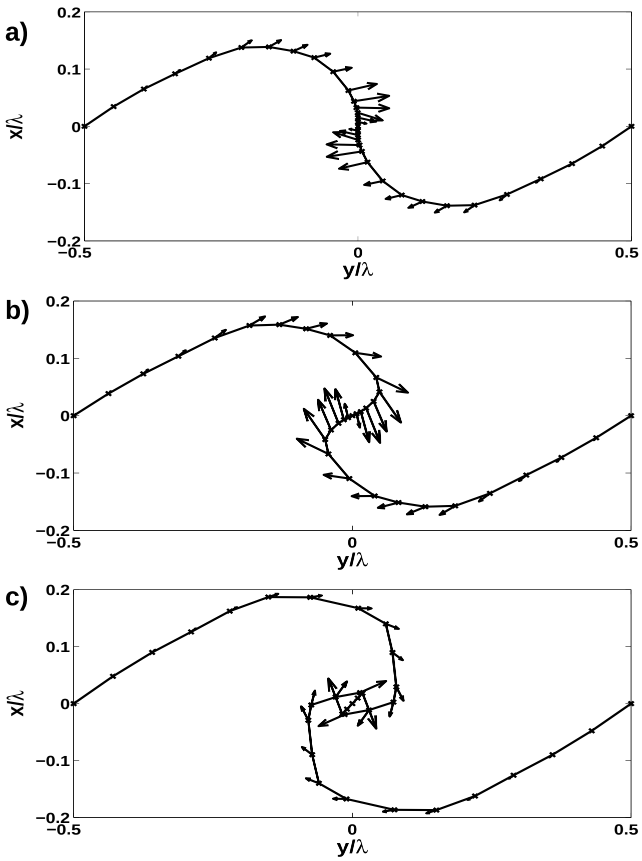

Fig. 3. Development of the Kelvin-Helmholtz instability in the equatorial plane.

4. Results of Stability Analysis

We assumed that K-H instability developed as a consequence to the preexisting FLR around 10 $\mathrm{R_E}$. We started with the magnitude of the resonance at 0.75 $\mathrm{R_E}$. The magnitude of the displacement defines the magnitude of the velocity shear. Then we assumed that there is a perturbation in the radial direction that will initiate vortex development (Fig. 3). For the stability analysis we have chosen stages a,c, and d of the vortex. Then we repeated tests with the initial magnitude of the FLR at 1 $\mathrm{R_E}$. Presence of the K-H instability leads to broadening of the disturbed region, propagating the perturbation further away from the resonance site. The initial setting (magnitude of resonance 0.75 $\mathrm{R_E}$) ensures that the initial energy density is in the linear regime. Just as in the case of a pure FLR type of displacement, for the 4:30 UT configuration the dominance of the second order was followed directly by the dominance of the fourth order term as the vortex development progresses. It means that any instability growth will be saturated due to nonlinear effects. For the initial magnitude of the displacement at 1 $\mathrm{R_E}$ we obtained similar results, with the difference that the fourth order term was dominant from the onset of the K-H instability.

For the 4:35 UT configuration starting with the initial magnitude of the FLR's at 0.75 $\mathrm{R_E}$ ensures the initial energy density to be in the linear regime (dominant second order term). However, even for the initial stages of the K-H instability, the third order term is much more important than for the pure resonance-type displacement. Further development of the vortex leads to a strong dominance of the third order term. This means that the presence of the vortex further destabilizes the near-Earth plasma sheet. For the initial magnitude of the resonance of 1 $\mathrm{R_E}$, even the initial stages of the vortex development lead to a dominant third order term in the energy density. We obtained the strongest dominance of the third order term for stage (c) (Fig. 3) of the vortex development. Further wrapping of the vortex leads to dominance by the fourth order term which means that it provides a stabilizing effect on the system.

The results of the analysis of the 4:40 UT configuration were analogical to the results at later growth phase. The initially dominant second order term for early stages of the vortex development are followed by the dominance of the fourth order term. This confirms that at this stage the excess energy was already released and the system is back in the lower energy state.

Figure 4 shows results for the magnitude of the resonance set at 0.75 $\mathrm{R_E}$ for the most unstable stage of the vortex (Fig. 3c) for all four configurations. The cross-section is taken through the region of maximum gradient in the velocity. Parts (a) and (c), corresponding to 4:30 UT and 4:40 UT configurations respectively, show stable behavior (dominant 4th order term). Part (b), corresponding to 4:35 UT configuration is explosively unstable. For this configuration, the third order term is dominant. Thus, around 4:35 UT, just minutes prior the onset, the near Earth plasma sheet became explosively unstable, while during growth phase and the recovery phase the near-Earth plasma sheet was nonlinearly stable.

Since the K-H instability does not extract potential energy, and only redistributes the kinetic energy, the presence of the K-H instability alone is not able to explain the energy reconfiguration in the near-Earth plasma sheet. Note in Fig. 3, that the presence of vortex wrapping leads to a saturation of the magnitude of the displacement. [14] proposed that a coupling between K-H and ballooning modes could lead to a growth of the vortex and thus to reconfiguration of the energy in the region.

To test the effect of this scenario on the change in the stability properties, we have assumed an increase in the size of the vortex, and calculated the energy density for such configurations. Fig. 5 shows energy density terms for a vortex that grows to twice its original size. Parts a) and b) corresponds to vortex sizes of 1 $\mathrm{R_E}$ and 2 $\mathrm{R_E}$. As the vortex grows, the fourth order term in energy becomes dominant, suggesting nonlinear saturation of the instability. This result agrees with the scenario of reconfiguration of energy due to K-H ballooning coupling and is consistent with the computational model of the shear-flow ballooning instabilities and observations of auroral arcs in [14] and [15].

To summarize, the development of the K-H instability from the velocity shear due to field line resonance provided similar stability properties for various stages of the Feb. 9, 1995 substorm as did the field line resonance alone. This suggests that these general stability results are not dependent on the type of the displacement, and any realistic displacement yields to the explosively unstable near-Earth plasma sheet minutes prior to the onset while it remained stable during the most of the growth phase.

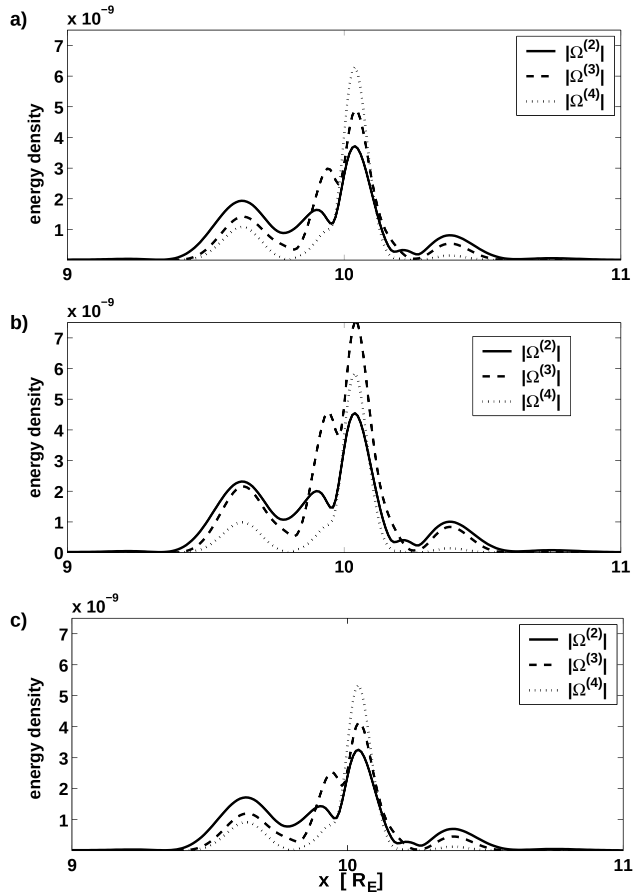

Fig. 4. Potential energy density in the case of a Kelvin-Helmholtz instability in the equatorial plane. The magnitude of the initial FLR was 0.75 Re.

5. Conclusions

We improved previous stability analysis by including a possibility of the development of a Kelvin-Helmholtz vortex in the system and analyzing its influence on the stability of the system. Our results suggest that during the stable state of a substorm the presence of the K-H vortex does not influence the general stability of the system. Since there is no available free energy, such a vortex must be saturated. In the case of unstable configuration during the onset, the presence of the K-H instability can cause a faster initiation of the explosive instability due to enhanced gradients in plasma displacement. However, we need to note that the K-H instability can be initiated in the presence of any shear in velocity and does not have to be tied to the presence of FLRs. On the other hand, if there is resonance present, it is likely that the K-H instability will appear. We can conclude that for the studied event, the transition between stable and unstable configurations corresponds to the time of onset no matter what the displacement is. The K-H instability alone cannot extract potential energy from the system. It only transforms different forms of kinetic energy. Therefore, we have also investigated what happens if the vortex grows due to coupling with ballooning modes. It appears that the growth of the vortex might eventually lead to reconfiguration of the energy and the saturation of the instability.

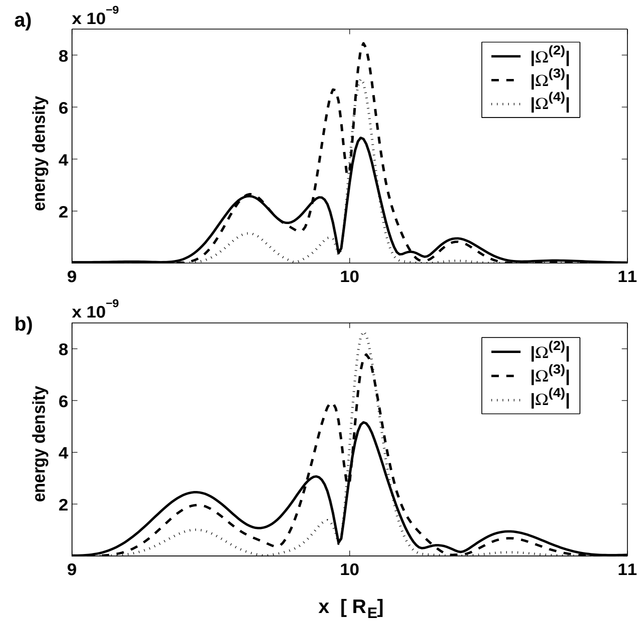

Fig. 5. Potential energy density for the 4:35 UT configuration for the K-H ballooning type of the displacement.

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References

1. Blanchard, G. T., L. R. Lyons, J. C. Samson, and F. J. Rich, Locating the polar cap boundary from observations of 6300 A emission, *J. Geophys. Res.*, 100, 7855, 1995.
2. Büchner, J., and L. M. Zelenyi, Chaotization of the electron motion as the cause of an internal magnetotail instability and the substorm onset, *J. Geophys. Res.*, 92, 13,456, 1987.
3. Chandrasekhar, S., *Hydrodynamic and hydromagnetic stability*, Oxford Press, 1961
4. Dobias, P., and J. C. Samson, Nonlinear instabilities in magnetized plasmas: A geometrical treatment, *Can. J. Phys*, 82, 593, 2004.
5. Dobias, P., I. O. Voronkov, and J. C. Samson, On nonlinear plasma instabilities during the expansive phase onset, *Phys. Plasmas*, 11, 2046, 2004a.
6. Dobias, P., J. A. Wanliss, I. O. Voronkov, and J. C. Samson, On the stability of the near-Earth magnetotail at growth phase, in: *Proceedings of the 7th International Conference on Substorms*, pg. 152, 2004b.
7. Fong, B. H., S. C. Cowley, O. A. Hurricane, Metastability of magnetically confined plasmas, *Phys. Rev. Let.*, 82, 4651, 1999.
8. Lyons, L. R., I. O. Voronkov, E. F. Donovan, and E. Zesta, Relation of substorm breakup arc to other growth phase auroral arcs, *J. Geoph. Res.*, 107, 1390, 2002.
9. Pfirsch, D., and R. N. Sudan, Nonlinear ideal magnetohydrodynamic instabilities, *Phys. Fluids*, 7, 2052, 1993.
10. Rosenhead, L, Formation of vortices from a surface of discontinuity, *Proc. Roy. Soc. A*, 134, 170, 1931.

11. Samson, J. C., L. L. Cogger, and Q. Pao, Observations of field line resonances, auroral arcs, and auroral vortex structures, *J. Geophys. Res.*, 101, 17373, 1996.
12. Samson, J. C., and P. Dobias, Explosive instabilities and substorm intensification in the near-Earth magnetotail, in: *Multiscale Coupling of Sun-Earth Processes*, edited by A.T.Y. Lui, Y. Kamide, and G. Consolini, 2004.
13. Samson, J. C., R. Rankin, and V. T. Tikhonchuk, Optical signatures of auroral arcs produced by field line resonances; comparison with satellite observations and modeling, *Annales Geophysicae*, 21, 933, 2003.
14. Voronkov, I., R. Rankin, P. Frytz, V. T. Tikhonchuk, and J.C. Samson, Coupling of shear flow and pressure gradient instabilities, *J. Geoph. Res.*, 102, 9639, 1997.
15. Voronkov, I., E. F. Donovan, B. J. Jackel, and J. C. Samson,Large-scale vortex dynamics in the evening and midnight auroral zone: Observations and simulations, *J. Geophys. Res.*, 105, 18,505, 2000.
16. Wanliss, J. A., J. C. Samson, and E. Friedrich, On the use of photometer data to map dynamics of the magnetotail current sheet during substorm growth phase, *J. Geophys. Res.*, 105, 27,673, 2000.
17. Wanliss, J. A., R. Rankin, J. C. Samson, and V. T. Tikhonchuk, Field line resonances in a stretched magnetotail: CANOPUS optical and magnetometer observations, *J. Geophys. Res.*, 107, 10.1029, 2002.
18. Zelenyi, L., A. Galeev, and C. F. Kennel, Ion precipitation from the inner plasma sheet due to stochastic diffusion, *J. Geophys. Res.*, 95, 3,871, 1990.